Multifractality and $1/f$ noise in the two-component random resistor network

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The two-component random resistor network, i.e., the network composed of conducting and insulating bonds, both with finite values of conductance (g_i and g_c), is analyzed. Based on the general scaling assumption, a single crossover exponent for small value of $h=g_i/g_c$ for all multifractal moments of current or voltage distributions is found not only in $d=2$ dimensions. This allows us to describe the behavior of $1/f$ noise of the two-component random resistor network over the entire region of concentration p of the conducting component: Three pictures of the relative noise intensity S versus the concentration p are admissible, depending on the ratio of the microscopic-$1/f$-noise intensities of the components.

I. INTRODUCTION

The $1/f$ noise, which arises from microscopic resistance fluctuations, in random resistor networks (RRN's) is closely related to the fourth moment of the current distribution within the network.[1,2] From this, and because of connections to some structural properties of percolating cluster, the question of "how the moments of current and voltage distributions scale in fractal and homogeneous regions" seems to be important.[1–4] Recently, the problem has been extended to the two-component random resistor network, i.e., the network containing two types of bonds, both of finite value of conductance.[5] For such a network the infinite number (one for each moment) of exponents describing the crossover from fractal to homogeneous behavior has been suggested.[5] In this paper, the above result is not confirmed. We deal with the infinite two-component network and consider the crossover exponents associated with a ratio of conductances of insulating to conducting components (which are trivially related to these mentioned above). We show that these exponents take only one value $1/(t+q)$ for all multifractal moments of current and voltage distributions in conducting and insulating phases. Upon this conclusion the behavior of $1/f$ noise in the two-component RRN above, at, and below the percolation threshold is reviewed. Existence of the two noise critical exponents $w=\kappa'+2q+2t$ and $w'=\kappa+2q+2t$ indicated recently by Morozovsky and Snarsky[6] and by Tremblay, Fourcade, and Breton[7] is confirmed and, in addition, conditions of their observability are established. The scaling function forms for moments of a distribution of power dissipated in the network is also proposed.

II. MULTIFRACTAL MOMENTS OF CURRENT AND VOLTAGE DISTRIBUTIONS IN THE TWO-COMPONENT RRN

Let us consider the random resistor network in which the effect, important from a practical point of view, of the nonzero conductance of the insulating phase is taken into account. In this network, the ratio of "poor" g_i and "good" g_c conductances, which form the whole network, is given by a small parameter $h=g_i/g_c$. Two conductances, g_i and g_c, occupy bonds of d-dimensional lattice randomly with probabilities $1-p$ and p, respectively. For such a network, the moments of current, M_n, and voltage, W_n, distributions can be defined separately for the insulating (i) and conducting (c) bonds:

$$M_{in}=\sum_{\alpha\in i}\left[\frac{I_\alpha}{I}\right]^{2n}, \tag{1a}$$

$$W_{in}=\sum_{\alpha\in i}\left[\frac{V_\alpha}{V}\right]^{2n}, \tag{1b}$$

$$M_{cn}=\sum_{\alpha\in c}\left[\frac{I_\alpha}{I}\right]^{2n}, \tag{1c}$$

$$W_{cn}=\sum_{\alpha\in c}\left[\frac{V_\alpha}{V}\right]^{2n}, \tag{1d}$$

where I_α (V_α) denotes current (voltage) in bond α [belonging to either the (i) or (c) phase] after the external current I (voltage V) is supplied to the network. All of the next results are based on the natural assumption that the quantities defined above are generalized homogeneous functions in a neighborhood of the point $h=0$, $\varepsilon=p-p_c=0$, i.e., near the percolation threshold p_c. The following relations are important

$$W_{in}=\left[\frac{G}{g_i}\right]^{2n}M_{in}, \tag{2a}$$

$$W_{cn}=\left[\frac{G}{g_c}\right]^{2n}M_{cn}, \tag{2b}$$

where G denotes the overall conductance of the network ($G=g_iW_{i1}+g_cW_{c1}$).

Some features of defined quantities are well known: For $h\to 0$, $W_{cn}\sim\varepsilon^{t_n}$, $M_{cn}\sim\varepsilon^{t_n-2nt}$ if $\varepsilon>0$ and $M_{in}\sim|\varepsilon|^{q_n}$, $W_{in}\sim|\varepsilon|^{q_n-2nq}$ if $\varepsilon<0$, where exponents $t=t_1$ and $q=q_1$ describe the ε dependence of the network conductance: $G \sim g_c \varepsilon^t$ for $\varepsilon > 0$ and $G \sim g_i |\varepsilon|^{-q}$ for $\varepsilon < 0$. The exponents t_n and q_n are simply related to infinite sets of multifractal exponents $p(2n)$ and $\tilde{\zeta}'(2n)$ introduced by de Arcangelis, Redner, and Coniglio[3,4] or to exponents x_n of Rammal, Tannous, Breton, and Tremblay:[2]

$$t_n = (d-2n)\nu + p(2n) = 2nt + \nu[x_n + d - 2n(d-1)] , \quad (3a)$$

$$q_n = 2nq - \nu[\tilde{\zeta}'(2n) + 2n - d] , \quad (3b)$$

where ν denotes the correlation length exponent. These sets of exponents describe dependence of moments M_n and W_n on the lattice size L at the percolation threshold in two limiting cases i.e., for random resistor networks ($g_i = 0$) and for random resistor superconductor networks (RRSN's) ($g_c = \infty$):

$$M_n = M_{cn} \sim L^{-x_n} \quad \text{for RRN's} , \quad (4a)$$

$$W_n = W_{cn} \sim L^{-p(2n)/\nu} \quad \text{for RRN's} , \quad (4b)$$

$$W_n = W_{in} \sim L^{\tilde{\zeta}'(2n)} \quad \text{for RRSN's} . \quad (4c)$$

The number of introduced exponents have been numerically estimated.[2–4,7,8]

III. SCALING APPROACH TO MULTIFRACTAL MOMENTS OF CURRENT AND VOLTAGE DISTRIBUTIONS

Let us make the usual scaling hypothesis for each of the moments in Eq. (1). First let us draw our attention to the insulating phase:

$$M_{in} = M_{in}(\varepsilon, h) = \lambda^{\alpha} M_{in}(\varepsilon/\lambda^{\beta}, h/\lambda) = h^{\alpha} m_{in}(\varepsilon/h^{\beta}) , \quad (5a)$$

$$W_{in} = W_{in}(\varepsilon, h) = \lambda^{\gamma} W_{in}(\varepsilon/\lambda^{\delta}, h/\lambda) = h^{\gamma} w_{in}(\varepsilon/h^{\delta}) , \quad (5b)$$

where λ is the usual scaling parameter. The exponents α, β, γ, and δ can be easily determined: For $\varepsilon < 0$ moments, M_{in} and W_{in} must behave as $|\varepsilon|^{q_n}$ and $|\varepsilon|^{q_n - 2nq}$, respectively. Consequently, $m_{in}(x) \sim |x|^{\alpha/\beta}$ and $w_{in}(x) \sim |x|^{\gamma/\delta}$ as $x \to -\infty$. Thus, the relations

$$\frac{\alpha}{\beta} = q_n , \quad (6a)$$

$$\frac{\gamma}{\delta} = q_n - 2nq , \quad (6b)$$

must be fulfilled. Similarly, for $\varepsilon = 0$, M_{in} and W_{in} scale with h as h^{α} and h^{γ}, respectively. Taking into account the h dependence of the lattice conductance G at the percolation threshold[9] ($\varepsilon = 0$)

$$G \sim g_i h^{-q/(t+q)} , \quad (7)$$

together with Eq. (2a), we obtain

$$\gamma = \alpha - \frac{2nq}{t+q} . \quad (6c)$$

To derive the above-threshold behavior of M_{in}, note that, for $\varepsilon > 0$, moments M_{in} should scale as h^{2n}. It is because, in this region, all nonzero currents I_{α} of bonds α belonging to the insulating phase scale as

$$I_{\alpha} = g_i V_{\alpha} \sim g_i V \sim g_i I/G \sim g_i I/(g_c \varepsilon^t) \sim hI$$

[see Eq. (2a)] for $\varepsilon > 0$. Immediately we have $m_{in}(x) \sim x^{(\alpha - 2n)/\beta}$ and $w_{in}(x) \sim x^{\gamma/\delta}$ as $x \to +\infty$ and

$$M_{in} \sim h^{2n} \varepsilon^{(\alpha-2n)/\beta} , \quad (8)$$

$$W_{in} \sim h^0 \varepsilon^{\gamma/\delta} , \quad (9)$$

for $\varepsilon > 0$. Now, looking again at Eq. (2a), we get

$$\frac{\gamma}{\delta} = 2nt + \frac{\alpha - 2n}{\beta} , \quad (6d)$$

which at last allows us to determine the exponents α, β, γ, and δ:

$$\beta = \delta = \frac{1}{t+q} , \quad (6e)$$

$$\alpha = \frac{q_n}{t+q} , \quad (6f)$$

$$\gamma = \frac{q_n - 2nq}{t+q} . \quad (6g)$$

The most important conclusion is that the crossover exponents δ, β take only one value for all multifractal moments M_{in} and W_{in}, i.e. those relevant to RRSN [see Eq. (4c)]. This is in contradiction to the result of de Arcangelis and Coniglio[5] who predicted different values of crossover exponents for different moments. The second important result is that multifractal exponents describing current or voltage distributions in the insulating phase above the percolation threshold are simply related [see Eqs. (8), (9), and (6e)–(6g)] to those of the below-threshold distributions. It can by utilized in analyzing RRN's with a small, but nonzero, conductance of the insulating phase.

In the same way, we can scale the multifractal moments M_{cn} and W_{cn} in the conducting phase. In this phase, voltages V_{α} scale as

$$V_{\alpha} = I_{\alpha}/g_c \sim I/g_c \sim VG/g_c \sim V g_i |\varepsilon|^{-q}/g_c \sim hV$$

for $\varepsilon < 0$. This leads to the scaling forms for moments M_{cn} and W_{cn}: Following the steps from Eq. (5a) to Eq. (6g), we get

$$W_{cn} = h^{t_n/(t+q)} w_{cn}(\varepsilon/h^{1/(t+q)}) , \quad (10)$$

where the scaling function $w_{cn}(x) \sim x^{t_n}$ as $x \to \infty$ and $w_{cn}(x) \sim |x|^{t_n - 2n(t+q)}$ as $x \to -\infty$. Thus, the conclusions derived above can be extended: Multifractal moments in the conducting phase have a single crossover exponent (the same as moments in the insulating phase have). Exponents describing scaling of moments in the conducting phase below the percolation threshold are related to those of the above-threshold scaling. It is important when analyzing RRSN's with a very large but finite conductance of the superconducting phase.

IV. MOMENTS OF DISTRIBUTION OF POWER DISSIPATED IN THE NETWORK

The moments of power dissipated in the network driven by the unit voltage $V=1$ are given by

$$P_n = g_i^n W_{in} + g_c^n W_{cn} = g_c^n[h^n h^{(q_n-2nq)/(t+q)} w_{in}(\varepsilon/h^{1/(t+q)}) + h^{t_n/(t+q)} w_{cn}(\varepsilon/h^{1/(t+q)})] \,. \quad (11)$$

This can be reduced to the simpler scaling form

$$P_n \sim h^u[f_1(\varepsilon/h^{1/(t+q)}) + f_2(\varepsilon/h^{1/(t+q)})] \quad (12)$$

[with $u=t/(t+q)$] only for $n=1$ as was done by a number of authors,[9] but it cannot be for the larger values of n as it has been recently proposed by de Arcangelis and Coniglio.[5]

V. 1/f NOISE ABOVE THE PERCOLATION THRESHOLD

The macroscopic conductance fluctuations (δG) of network conductance G can be expressed, according to the Rammal-Tannous-Tremblay[1] formula, in terms of microscopic conductance fluctuations δg_α of bond conductance g_α. For the two-component network and spatially uncorrelated fluctuations we have

$$S_G = s_{gi} W_{i2} + s_{gc} W_{c2} \,, \quad (13)$$

where $S_G = \langle(\delta G)^2\rangle$, $s_{gi} = \langle(\delta g_i)^2\rangle$, and $s_{gc} = \langle(\delta g_c)^2\rangle$ denote the conductance noise intensities of the whole network, and insulating and conducting bonds, respectively. Above the percolation threshold, p_c, or, more precisely, for $\varepsilon/h^{1/(t+q)} \gg 1$, Eq. (13) takes the form [see Eqs. (9) and (10)]

$$S_G \sim s_{gi}\varepsilon^{-\kappa'-2q} + s_{gc}\varepsilon^{-\kappa+2t} \,, \quad (14)$$

where the well-known noise exponents[10] κ' and[1,2,7,8,10] κ are related to the exponents $q_2 = -\kappa'+2q$ and $t_2 = -\kappa+2t$, respectively. A more universal quantity, the lattice relative noise $S = S_G/G^2$ above p_c, depends on ε as follows:

$$S \sim s_i h^2 \varepsilon^{-\kappa'-2t-2q} + s_c \varepsilon^{-\kappa} \,, \quad (15)$$

where $s_i = s_{gi}/g_i^2$ and $s_c = s_{gc}/g_c^2$ denote the bond relative noise intensities of the insulating and conducting bonds, respectively. It means that a much larger (than κ) value

$$w = \kappa' + 2t + 2q \quad (16)$$

of noise exponent is observed near the percolation threshold if the first term of Eq. (15) prevails over the second one. It occurs when the ratio of relative noise levels of insulating to conducting phases is sufficiently large. To be more precise, for

$$\frac{s_i}{s_c} \gg h^{(\kappa'-\kappa)/(t+q)} \,, \quad (17)$$

the exponent w can be observed in the region

$$h^{1/(t+q)} \ll \varepsilon \ll \left[\frac{s_i}{s_c} h^2\right]^{1/(w-\kappa)} \,. \quad (18)$$

The obtained result was indicated earlier by Morozovsky and Snarsky[6] and Tremblay, Fourcade, and Breton.[7] Morozovsky and Snarsky predicted the existence of the exponent w considering a certain topological model of the two-phase random medium. Tremblay, Fourcade, and Breton[7] performed finite-size-scaling simulations of the ($d=2$) dimensional two-component random resistor network in which the condition Eq. (17) was fulfilled with a large over plus. They computed the value 6.5 ± 0.3 of the noise critical exponent which is in excellent agreement with our prediction, Eq. (16), which yields $w=6.32\pm0.03$ and $w=6.1\pm0.6$ in $d=2$ and $d=3$, respectively, if the

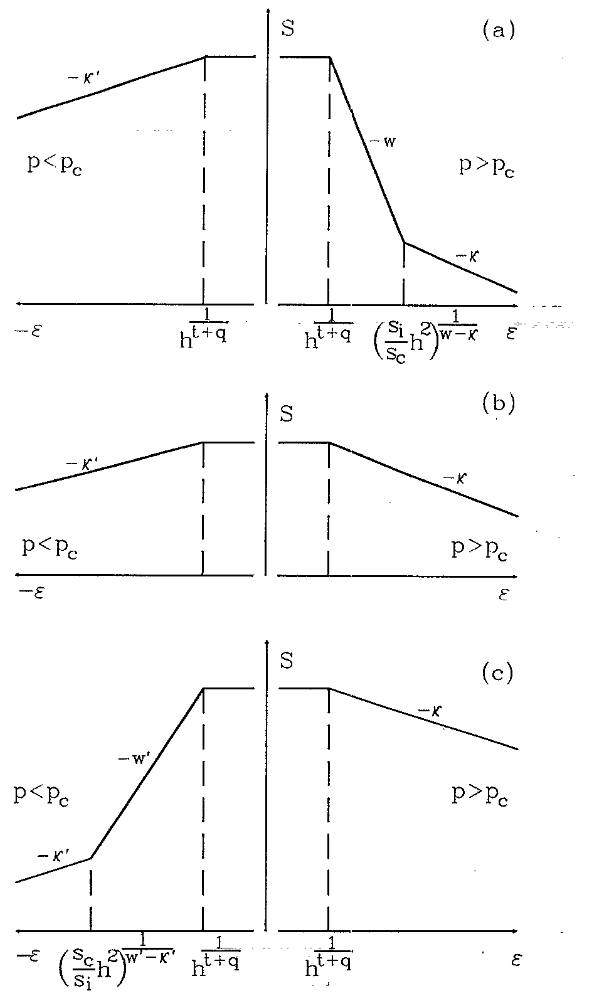

FIG. 1. Double logarithmic plot of the lattice 1/f noise relative intensity S as a function of $\varepsilon = p - p_c$, where p is the concentration of the metallic component in the two-component random resistor network, and p_c is the percolation threshold value. Three cases are possible depending on the relation between the ratio of bond conductances $h = g_i/g_c$ and the ratio of bond 1/f noise relative intensities s_i/s_c. (a) $s_i/s_c \gg h^{(\kappa'-\kappa)/(t+q)}$, exponents κ, w, and κ' can be detected; (b) $s_i/s_c \approx h^{(\kappa'-\kappa)/(t+q)}$, exponents κ and κ' can be detected; (c) $s_i/s_c \ll h^{(\kappa'-\kappa)/(t+q)}$, exponents κ, w', and κ' can be detected.

more recent estimations of exponents κ', t, and q are utilized.[2,7,8,10-13]

VI. $1/f$ NOISE AT THE PERCOLATION THRESHOLD

At the percolation threshold, the lattice relative noise S can be described by inserting Eqs. (5b) and (10) into Eq. (13), putting $\varepsilon=0$, and dividing by the square of the lattice conductance [Eq. (7)]:

$$S \sim s_i h^{-\kappa'/(t+q)} + s_c h^{-\kappa/(t+q)} . \tag{19}$$

This relation can by utilized to describe the noise of the systems treated as "working" at the critical point.

VII. $1/f$ NOISE BELOW THE PERCOLATION THRESHOLD

Eventually the derived scaling forms for the multifractal moments in Eqs. (1) make possible the determination of $1/f$ noise behavior below the percolation threshold. Putting Eqs. (5b) and (10) into Eq. (13) with $\varepsilon<0$ and dividing by $G^2 \sim |\varepsilon|^{-2q}$, we have

$$S \sim s_i |\varepsilon|^{-\kappa'} + s_c h^2 |\varepsilon|^{-\kappa-2q-2t} . \tag{20}$$

It means that the critical exponent $w'=\kappa+2q+2t$ (much larger then κ') related to the above threshold noise exponent κ can be observed below p_c when the condition Eq. (17) is fulfilled but in the opposite direction (i.e., with the $\ll$ sign). If not, the usual well-known exponent κ' describes the below-threshold behavior in the whole range of $\varepsilon<0$. The estimated values of the exponent w' are 6.32 ± 0.03 and 6.94 ± 0.4 for $d=2$ and $d=3$, respectively. Summing up the three cases of the $1/f$ noise relative intensity S versus concentration $\varepsilon=p-p_c$ are possible depending on whether the condition Eq. (17) is fulfilled (see Fig. 1).

VIII. CONCLUSIONS

We have supplied some arguments that there is a single crossover exponent $\beta=1/(t+q)$ associated with the small finite ratio h of the conductance of insulating to conducting bonds for all multifractal moments not only in two but also in higher dimensions. The only assumption used was that the functions describing multifractal moments are generalized homogeneous functions. Only recently have I received a preprint by Tremblay, Albinet, and Tremblay[14] which deals with the same problem in quite a similar way. They also propose the scaling approach to analyzed problem. However, they only study percolating networks, i.e., the $\varepsilon>0$ region. Arguments that they used to arrive at particular scaling forms of multifractal moments seem to be different from ours as well. Numerical simulations which they perform confirm that there is a single crossover exponents for $n=1,2,3$ multifractal moments in $d=2$ and $d=3$ dimensions.

Apart from this, we have proposed the scaling forms for moments of power dissipated in the network and reviewed the $1/f$ noise behavior in the two-component random resistor network. The latter is described by Eqs. (15)–(20), which agree with those obtained by Morozovsky and Snarsky.[7] Our approach, however, seems to be more general than that of Morozovsky and Snarsky[7] because of no restriction on the topology of the utilized model. The topological model used by them loses, in fact, the effect of multifractality of current distribution in the percolating cluster—exponents t_n and q_n can be computed in terms of exponents t, q, ν, d, and n only.

One of the unusual conclusions is that the condition Eq. (17) necessary to observe $1/f$ noise coming from insulating phase, above p_c, is not as strong as it was suggested.[7] We predict approximately $s_i \gg s_c$ in $d=2$ or $s_i \gg s_c h^{-0.3}$ in $d=3$ as sufficient enough to observe the exponent w, above p_c. Numerical checking of these predictions will be interesting.

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