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$1/f$ noise in binary random mixtures

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Novel numerical results for a two-component random resistor network as a model of a mixture with non-zero conductivity of the insulating phase are presented. They confirm that the values of noise-critical exponent depend on the noise intensity of the components of the mixture.

The scope of the paper is to study $1/f$ noise in a real insulator–conductor mixture, i.e. a mixture composed of conducting and insulating components both of non-zero conductivity. Extensive investigation of this are in progress (Morozovsky and Snarsky 1989, Tremblay *et al.* 1989, Tremblay *et al.* 1991, Kolek 1992). To study such a mixture a random resistor network (RRN) is often used as a model of the internal topology of a real composite. The RRN is a d-dimensional lattice composed of bonds occupied by two finite conductances g_i and g_c ($g_i \ll g_c$). Conductances g_i and g_c which model insulating and conducting phases of a mixture occupy all bonds of the lattice randomly with probability $1-p$ and p, respectively. Thus p is related to the concentration of a conducting component of a mixture. As $1/f$ noise arises from microscopic resistance fluctuations, the $1/f$ noise microsources are associated with all bonds of the lattice. However their intensities are different for various phases. Namely for conductances of the insulating bonds, g_i, and conducting bonds, g_c, the relative power spectral densities of their microfluctuations are $s_i(f)=s_i/f$ and $s_c(f)=s_c/f$ respectively. For a model defined in this way the question arises as to how the RRN (mixture) overall $1/f$ noise relative intensity S depends on the concentration of a conducting phase p (i.e., on the composition of the mixture).

The answer can be given with the help of scaling theory applied to multifractal moments of current and voltage distributions in RRN (Kolek 1992). One of the most important results that has been derived, is that a new critical exponent, w, can describe the power law dependence of S above the percolation threshold p_c, i.e. $S \sim (p-p_c)^{-w}$, instead of the exponent κ, which was found (Rammal *et al.* 1985) for the ideal insulator–conductor composite ($g_i=0$), i.e. instead of $S \sim (p-p_c)^{-\kappa}$. The sufficient condition for the exponent w to be observed is

$$\frac{s_i}{s_c} \gg \left(\frac{g_i}{g_c}\right)^{(\kappa'-\kappa)/(t+q)} \qquad (1)$$

i.e. that an insulating phase of a mixture is much 'noisier' than a conducting one. κ' and q denote noise and conductivity critical exponents of the ideal-superconductor–conductor mixture ($g_c=\infty$) respectively, whereas t is the conductivity exponent of the ideal insulator–conductor mixture. It has also been found that the exponent w is combined from exponents κ', t and q: $w=\kappa'+2t+2q$. Thus w takes values much greater than κ: $w=6{\cdot}32\pm0{\cdot}03$ and $6{\cdot}1\pm0{\cdot}6$ compared to $\kappa=1{\cdot}12\pm0{\cdot}02$ (Rammal

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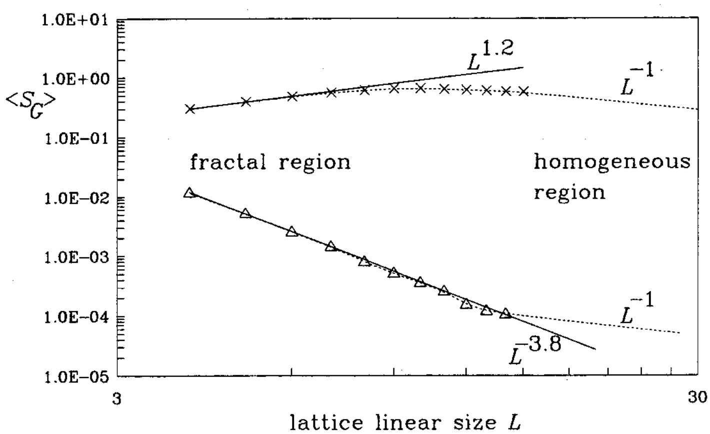
et al. 1985) and $1{\cdot}47 \pm 0{\cdot}04$ (Kolek and Kusy 1988) in $d=2$ and $d=3$ respectively. If this is true, anomalously large values of noise exponent sometimes measured in nature (see e.g. Garfunkel and Weissman 1985, Chen and Chou 1985) are still less surprising.

In this paper, new numerical results confirming scaling theory predictions cited above are supplied. Calculations were done on the simple cubic ($d=3$) RRN in which two kinds of bonds $g_c=1$, $s_c=1$ and $g_i=3\times10^{-4}$, $s_i=3{\cdot}3\times10^5$ were mixed. Thus the condition (1) was fulfilled ($((\kappa'-\kappa)/(t+q)\cong -0{\cdot}3$ for $d=3$). The conjugate gradient method was used to solve a matrix of Kirchhoff network equations and to find the distribution of bond voltages in RRN excited by an external voltage. Next the intensity of the conductance noise S_G was computed. It is related to S via $S_G=S\times G^2$, where $G\sim(p-p_c)^t$ is the conductance of RRN. Because of this, the exponents $-w+2t$ or $-\kappa+2t$ are expected to describe S_G vs. $p-p_c$, depending on whether condition (1) is fulfilled or not. To avoid large computational effort, finite size scaling was used instead of direct calculation of S_G vs. $p-p_c$: the average lattice conductance noise, $\langle S_G\rangle$, was computed as a function of lattice size L at $p=p_c$. (In fact higher order moments: $\langle S_G^n\rangle^{1/n}$ were computed to refer to the above threshold behaviour of RRN). Since in the homogeneous region S_G should vary with L as L^{-1} exponents z describing the size dependence of S_G in the fractal regime, i.e. $S_G\sim L^z$, should take: $z=d-4-(2t-w)/v=1{\cdot}4\pm0{\cdot}5$ or $z=d-4-(2t-\kappa)/v=-3{\cdot}75\pm0{\cdot}24$ if (1) is fulfilled or not. Above, v denotes the percolation correlation length exponent.

The results obtained are drawn in the figure together with the results for the RRN which did not fulfil (1): $g_c=1$, $s_c=1$ and $g_i=3\times10^{-4}$, $s_i=33$ were chosen in this case. The latter are presented for comparison. In these two cases, two different values of size dependence exponent z are detected in the fractal regime, i.e.: $z\cong1{\cdot}2$ for the lattice which fulfil condition (1) (the upper curve) and $z\cong-3{\cdot}8$ for the other. These values are in excellent agreement with our earlier predictions based on scaling theory (Kolek 1992).



Finite size scaling calculation of conductance noise S_G. Crosses refer to RRN composed of bonds $g_c=1$, $s_c=1$ and $g_i=3\times10^{-4}$, $s_i=3{\cdot}3\times10^5$. Triangles refer to RRN built from bonds $g_c=1$, $s_c=1$ and $g_i=3\times10^{-4}$, $s_i=33$. Slopes of the lines in fractal and homogeneous regions are indicated on the plots.

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