Excess 1/*f* noise in systems with an exponentially wide spectrum of resistances and dual universality of the percolation-like noise exponent

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The excess $1/f$ noise in a random lattice with bond resistances $r \sim \exp(-\lambda x)$, where x is a random variable and $\lambda \gg 1$, is studied theoretically. It is shown that if the correlation function $\{\delta r^2\} \sim r^{\theta+2}$, then the relative spectral density of the noise in the system is expressed as $C_e \sim \lambda^m \exp(-\lambda(1-p_c))$, where p_c is the percolation threshold and $m = \nu d$ (ν is the critical exponent of the correlation length and d is the dimensionality of the problem). It is hypothesized that the exponent m possesses a dual universality: It is independent of 1) the geometry of the lattice and 2) the θ-mechanism responsible for the generation of the local noise. Numerical modeling in a three-dimensional lattice gives $m = 2.3$ for $\theta = 1$ and $\theta = 0$, in agreement with the hypothesis. © *1996 American Institute of Physics.* [S0021-3640(96)01008-0]

A large number of problems reduce to the calculation of the effective properties of systems with an exponentially wide spectrum of resistances (ESR). For example, in the investigation of high-temperature hopping conductivity in weakly doped semiconductors it is necessary to determine the effective electrical conductivity σ_e of a random network with an exponentially wide spectrum of resistances — the so-called Miller–Abrahams network.[1] The resistance of the ith bond in this network can be chosen in the form

$$r_i = r_0 \exp(-\lambda x), \quad x \in [0,1], \quad \lambda \gg 1, \tag{1}$$

where x is a random variable with a smooth distribution $D(x)$ which is the same for all bonds.

As was shown in Refs. 1–4, there exists a method that reduces the problem of determining σ_e for the Miller–Abrahams network to a percolation problem. Neglecting the pre-exponential factor, this method leads to $\sigma_e = \sigma_0 \exp(\lambda x_c)$, where x_c is determined in terms of the percolation threshold of the standard two–phase problem on the same network

$$\int_0^1 D(x)dx = p_c, \quad \int_x^1 D(x)dx = p,$$

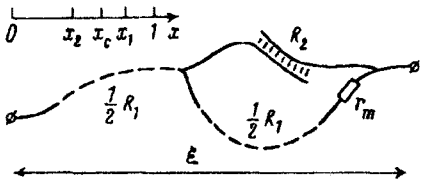

FIG. 1. Model of a percolation structure in the smearing region. R_1—resistance of ''simply connected bonds'' of the good-conductor phase; R_2—''simply disconnected''—parallel-connected bonds of the poor-conductor phase. The resistance r_m assumes with equal probability values from the smearing region.[17]

where p is a nominal concentration of the good-conductor phase. More detailed investigations of the dependence of σ_e on λ (Refs. 1 and 5–12) made it possible to find the percolation-like exponent y determining the behavior of the pre-exponential factor:

$$\sigma_e = \sigma_0 \lambda^{-y} \exp(\lambda x_c). \tag{2}$$

According to these investigations, the exponent y can be expressed in terms of the critical exponent $-\nu$, well known in the percolation theory, of the correlation length $y = \nu(d-2)$, where d is the dimension of the problem.

In Refs. 13 and 14 it was shown that to describe consistently the percolation-like behavior of the effective quantities, specifically σ_e in systems with ESR, it must be assumed that the system is in the so-called smearing region[15] (analog of the region of broadening of a phase transition in the theory of second-order phase transitions). In calculating the effective quantities it is necessary to use some model of the percolation structure in the smearing region. Such a model, which is an extension of models of the Nodes–Lins–Blobs type[5,6,11–16] to the smearing region, was proposed in Ref. 17 (see Fig. 1). In Ref. 18 this model was used in systems with ESR. According to this model, the percolation structure of a system with ESR consists of a bridge with resistance $R_1 \sim \langle r \rangle_1 N_1$, interlayers with resistance $R_2 \sim 1/(\langle 1/r \rangle_2 N_2)$, and a resistance r_m. Here $\langle r \rangle_1$ is the average resistance in the bridge, the average being taken over the interval of the random variable $x \in (x_1,1)$, and $\langle 1/r \rangle_2$ is the average in the interlayer (where in contrast to the bridge the resistances are connected in parallel) taken over the interval $x \in (0,x_2)$. The values of x_i are related with the size of the smearing region $\Delta - x_1 = x_c + (1-x_c)\Delta$ and $x_2 = x_c - (1-x_c)\Delta$.[14] In the smearing region the number of resistances in a bridge and an interlayer also depends on Δ: $N_i \sim \Delta^{-\alpha_i}$, where α_i are critical exponents which have different numerical values in different models. In two-phase percolation systems the choices $\alpha_1 = t - \nu(d-2)$ and $\alpha_2 = q + \nu(d-2)$,[19] where t and q are the critical exponents of the conductivity above and below the percolation threshold, gives good agreement between the experimental and numerical data. In Ref. 18 it was shown that for systems with ESR the choice $\alpha_1 = \alpha_2 = 1$, as done for two-phase systems in Refs. 5, 16, and 20–23, gives better agreement. For this reason, in what follows these values of α_i will be adopted everywhere and both values of α_i will be presented only in the table. The resistance $r_m = r_m(x)$ assumes with equal probability any value in the interval $x \in (x_2, x_1)$, corresponding to the smearing region in a two-phase system.

A characteristic of inhomogeneous media which is just as important as σ_e is the relative spectral density C_e of the excess $1/f$ noise. There are a large number of works devoted to the investigation of C_e in two-phase media near the percolation threshold (Ref. 6; see also the references cited in Ref. 14). The problem of determining C_e in macroscopically inhomogeneous media is formulated similarly to the σ_e problem—to determine C_e, expressed in terms of the second moment of the Joule-heat distribution, from prescribed local values $C(r)$:[24]

$$C_e=\frac{\langle C(\mathbf{r})(\mathbf{E}(\mathbf{r})\cdot\mathbf{j}(\mathbf{r}))^2\rangle}{(\langle\mathbf{E}(\mathbf{r})\rangle\cdot\langle\mathbf{j}(\mathbf{r})\rangle)^2}=\frac{\langle C(\mathbf{r})\sigma^2(\mathbf{E}(\mathbf{r})/\langle\mathbf{E}\rangle)^4\rangle}{\sigma_e^2}. \tag{3}$$

We note that since the local electric field normalized to the average value over the sample enters in the expression (3), C_e itself does not depend on the magnitude of the applied field. For what follows, it is necessary to make an assumption analogous to Hooge's hypothesis,[25] according to which $C=\kappa/\sigma^\theta$, where $\kappa\approx$ const is Hooge's universal constant, and it is assumed that $\sigma\sim n$ is the free-carrier density and $\theta=1$.

In Ref. 14 an expression was found for C_e for the case $\theta=1$ and a new (''noise'') percolation-like exponent in a system with ESR was calculated:

$$C_e\sim\sigma_e^{-1}\lambda^{2\nu}=\sigma_0^{-1}\exp(-\lambda x_c)\lambda^{m_1},\quad m_1=y+2\nu=\nu d,\quad \theta=1. \tag{4}$$

The calculation followed the scheme in Fig. 1, according to which the local values of the currents $j(r)$ and voltages $E(r)$ can be determined in the main elements of the percolation structure—bridge, interlayer, and resistance r_m. According to the percolation approach to the description of kinetic phenomena in media near the percolation threshold, the current density is highest in a bridge and the main voltage drop is across the interlayer, so that in calculating σ_e the other elements of the structure are neglected. In contrast to σ_e, the second, and not the first, moment of the Joule heat (3) enters in C_e, so that with even greater justification only the structural elements indicated in Fig. 1 need be considered in calculating C_e.

It was shown recently in Refs. 26 and 27 that in a number of cases the local values of C in tunnel junctions may not depend on σ, i.e. $C\approx\kappa/\sigma^\theta$ with $\theta=0$. In the model of a percolation structure in the smearing region, we found analytical expressions for C_e for arbitrary $0<\theta<2$ and performed numerical modeling for the two cases $\theta=1$ and $\theta=0$. According to our calculations, the expressions for C_e and for the noise percolation-like exponent m_θ have the form

$$C_e\sim(\sigma_0\exp(\lambda x_c)^{-\theta}\lambda^{m_\theta},\quad \text{and}\quad m_\theta=\nu d, \tag{5}$$

and m_θ does not depend on θ.

To check the values obtained for m_θ, a network with an ESR was modeled. In numerical simulations it is more convenient to employ the relative spectral density S_G of the excess $1/f$ noise of a volume V, which is related with the C_e by the relation $G^2C_e=VS_G$, where G is the conductance of the volume V. According to Eq. (4), $S_G=\Sigma S_{gi}(U_i/U)^4$, where U_i is the voltage drop on the ith bond, U is the total voltage drop, and S_{gi} is the noise of the ith bond ($S_{gi}=\{\delta g_i,\delta g_i\}$, where $\{\ldots\}$ denotes a time correlation function). Just as in the case of C_e defined in Eq. (3), the fourth power of

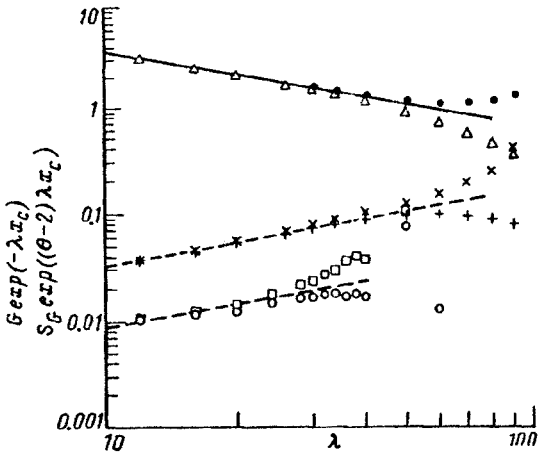

FIG. 2. Numerical modeling results. The conductance G (solid line) and noise S_G (dashed line) are plotted in double-logarithmic coordinates. The percolation-like exponents are determined by the least-squares method applied separately for the arithmetic (index a) and harmonic (index h) means over realizations of the random spread in the resistances: □) $G_a\exp(-\lambda x_c)$, Δ) $G_h\exp(-\lambda x_c)$, ∗) $S_{G_a}\exp(\theta-2)\lambda x_c|_{\theta=1}$, +) $S_{G_h}\exp(\theta-2)\lambda x_c|_{\theta=1}$, □) $S_{G_a}\exp(\theta-2)\lambda x_c|_{\theta=0}$, ○) $S_{G_h}\exp(\theta-2)\lambda x_c|_{\theta=0}$.

U_i does not mean that the intensity of $1/f$ noise is proportional to the fourth power of the voltage applied to the sample—only the relative value of the voltage drop enters in S_G.

At each step of the calculations a simple cubic lattice with dimension $L=a_0N$ (a_0 is the bond length) was constructed and one possible realization of the resistance distribution was examined. The solution of the system of Kirchhoff equations gave the voltage on all bonds in the lattice. This made it possible to determine the conductance G and the noise S_G for the entire system.

The data obtained are presented in Fig. 2; the slope determines the value of the percolation-like exponents y and $m-2y$. As one can see, a difference between the averages (arithmetic means G_a and S_{G_a} and harmonic means G_h and S_{G_h}) first appears at $\lambda\sim30$. This means that for $\lambda>30$ the system possesses a fractal structure. Indeed, for the value $N=15$ chosen (for $N>15$ the memory exceeds 6 MB, which is important for programming in Windows) the correlation length of the system $\xi\sim a_0\Delta^{-\nu}\sim a_0\lambda^{\nu}$ (Ref. 14) for $\lambda\approx40$ is already twice as large as the representative size of the system $\xi/a_0\approx30$. The numerical results presented in Table I show good agreement with the analytical expression $m_\theta=\nu d$.

It is interesting to note that although $C_e(\theta=1)$ differs exponentially from $C_e(\theta=0)$, for example, for $\lambda\approx30$ and $x_e=0.75$

$$C_e(\theta=0)/C_e(\theta=1)=e^{\lambda x_c}\sim10^9, \tag{6}$$

the exponent m_θ remains the same.

TABLE I. Comparison of the results of an analytical calculation of the percolation-like exponents y and m with data from numerical modeling.[†]

	y	m_1	m_1-2y	m_0	m_0-2y
$\alpha_1=\alpha_2=1$	0.89	2.67	0.89	2.67	0.89
$\alpha_1=t-\nu$ $\alpha_2=q+\nu$	0.56	2.30	1.18	2.03	0.84
Numerical modeling	0.6 ± 0.1* 0.76 ± 0.04	2.30	$0.78^{+0.09}_{-0.08}$	2.26	$0.74^{+0.3}_{-0.22}$

[†]The values obtained for the exponents in the model of a percolation structure in the smearing region are presented in the first two rows. The numerical values of the critical exponents are: $t=1.94$, $q=0.75$, and $\nu=0.89$. The value of y marked with the asterisk was obtained in Ref. 9.

In summary, it is possible to talk about a ''dual universality'' of the percolation-like exponent m. First of all, m, just as the critical exponents t, q, and ν, should not depend on the type of lattice (but only its dimension); second, as the calculations presented above for $0<\theta<2$ and the numerical modeling suggest, the exponent m does not depend on the choice of ''Hooge's hypothesis.''

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